



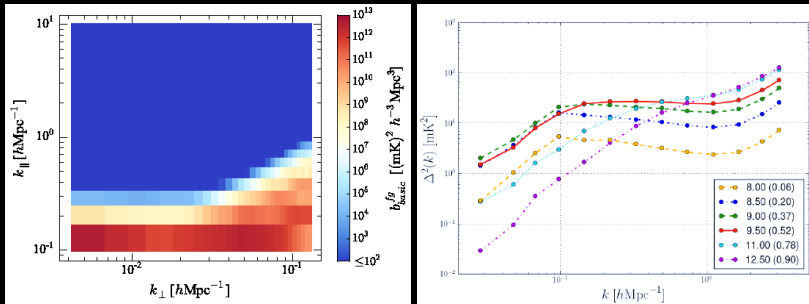
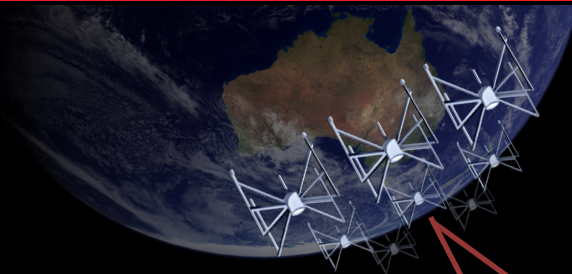
International  
Centre for  
Radio  
Astronomy  
Research

# Redundant Calibration: Breaking the constraints of limited sky information

**Ronniy Joseph**  [@astronniy](#)  
Cath Trott and Randall Wayth

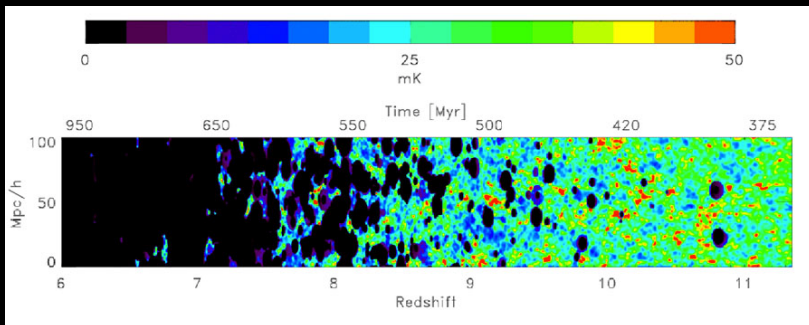
*IAUS333 – Peering towards Cosmic Dawn*

# Epoch of Reionisation

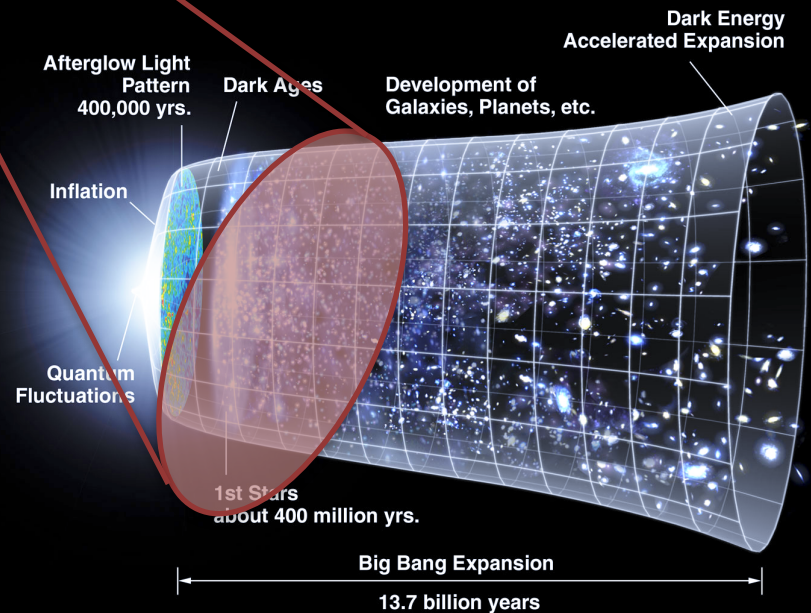


Liu, Parsons, Trott 2014

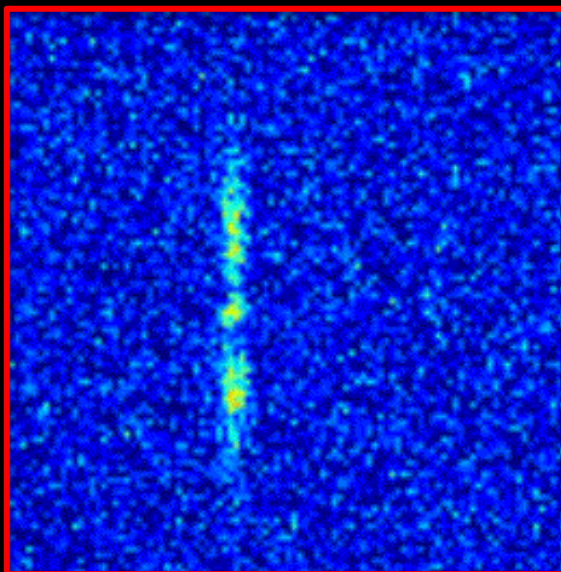
Pober et al. 2013



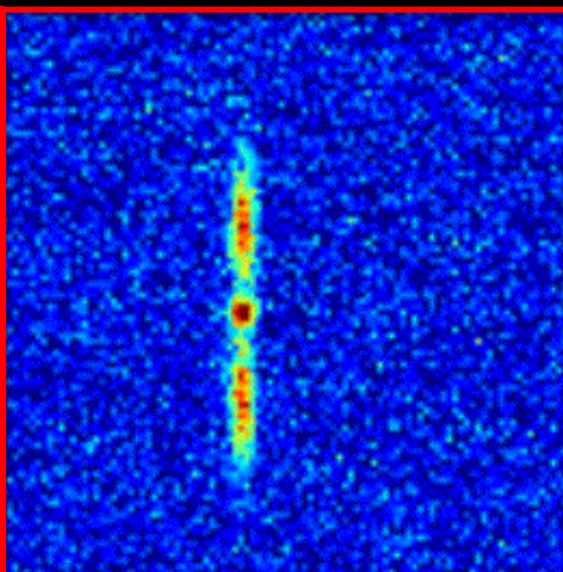
Thomas et. al 2009



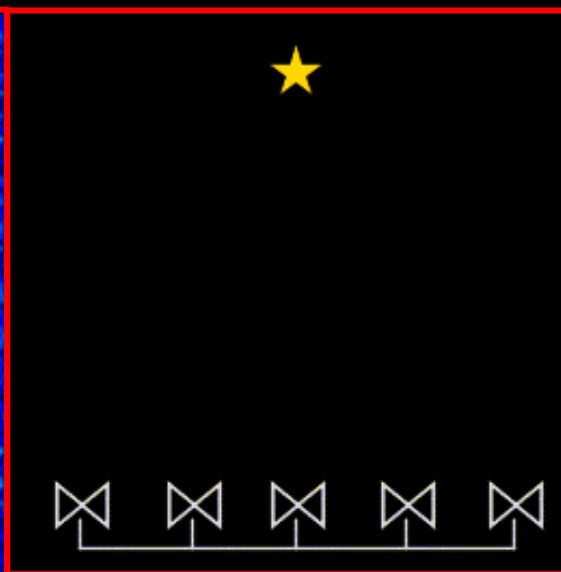
Uncorrected



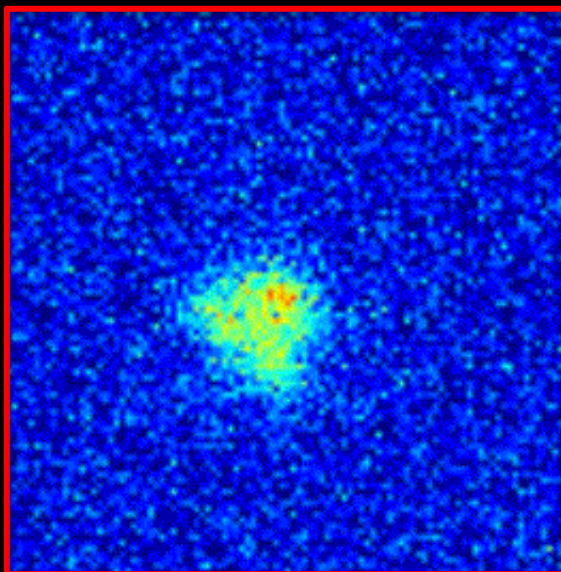
Corrected



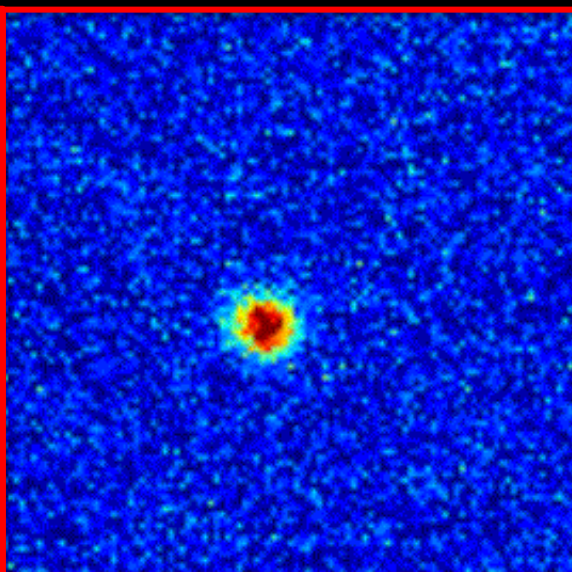
Modifying Antenna Response



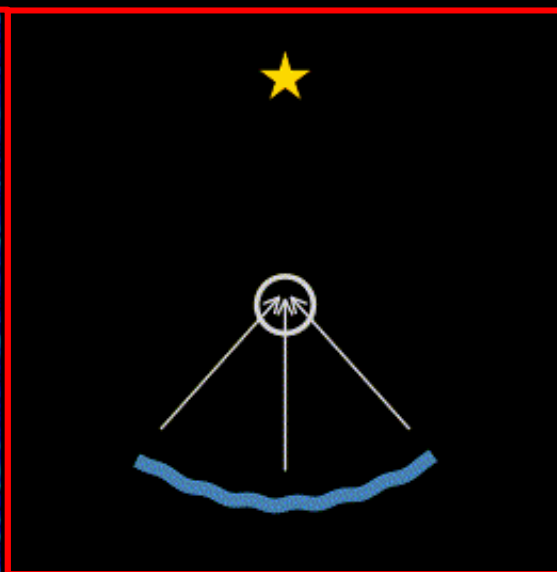
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Corrected

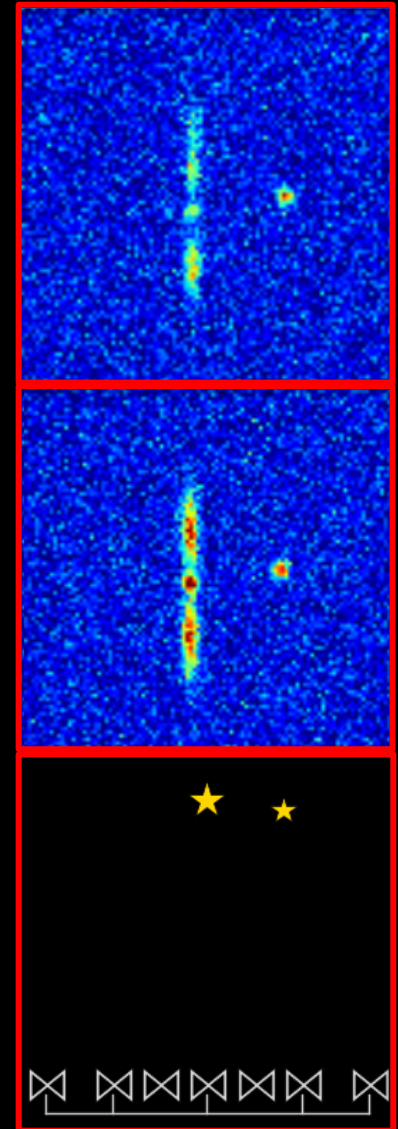


Deforming Mirror

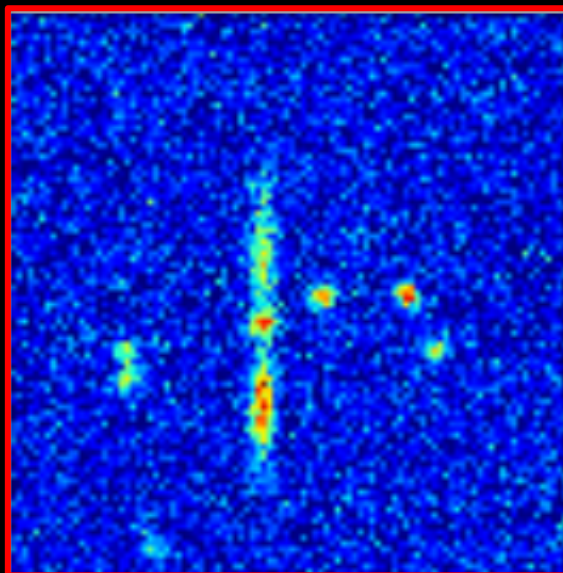




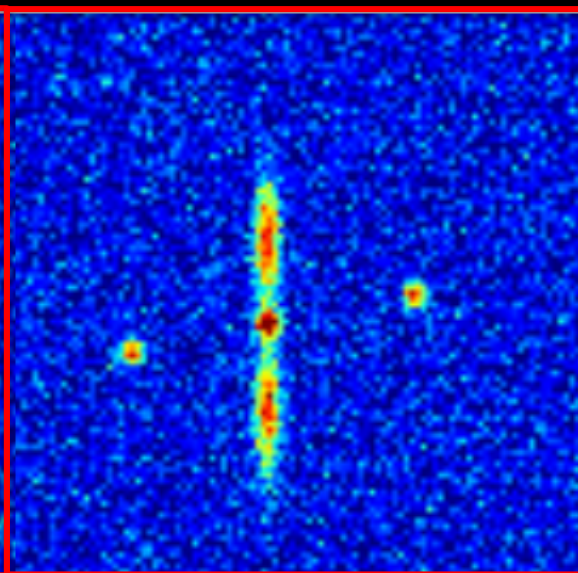
- Digital Correction
- Post-observation
- Requires **Models/Catalogues**



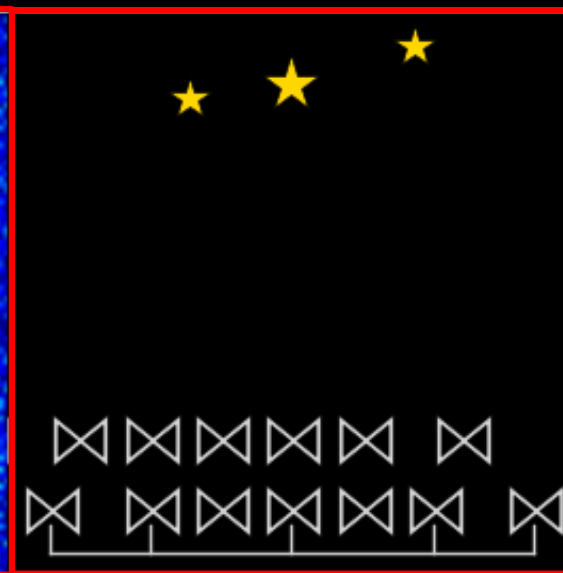
Uncorrected



Corrected



Modifying Antenna Response





# Self Calibration – Radio

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## Incomplete Models

- **Incorrect Calibration Solutions**
- **Image Artefacts** [Grobler et. al. 2016, Wijnholds et. al. 2016]

## Revise Calibration Strategies

- **Redundant Calibration**



# Interferometry Intermezzo

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<http://egg.astro.cornell.edu/alfalfa/ugrad/hidistrib.html>



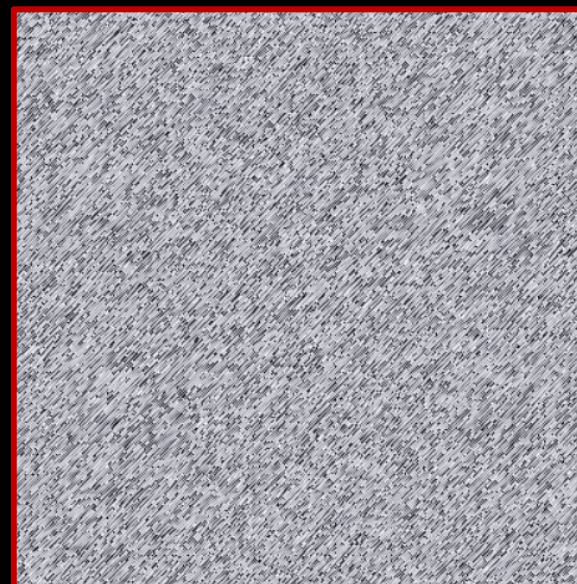


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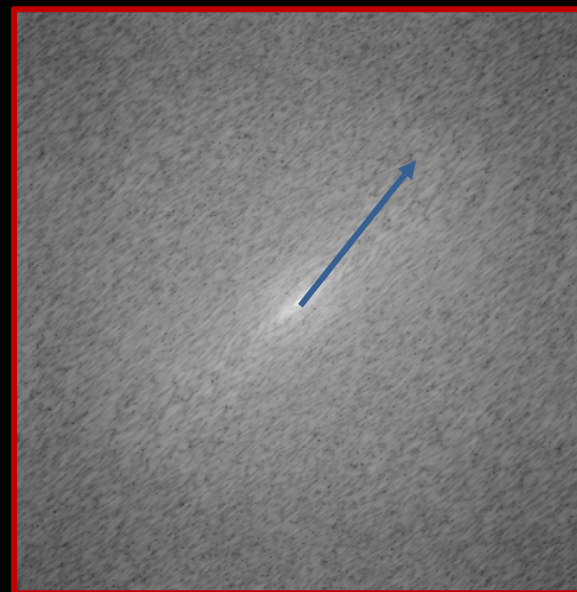
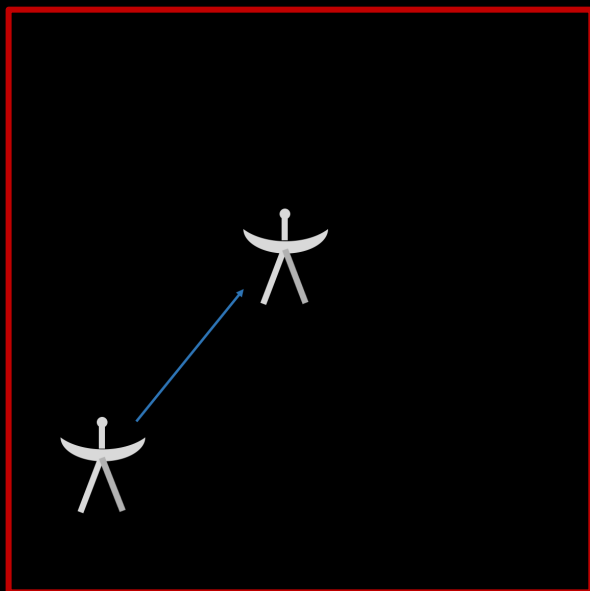
**Fourier Transform**



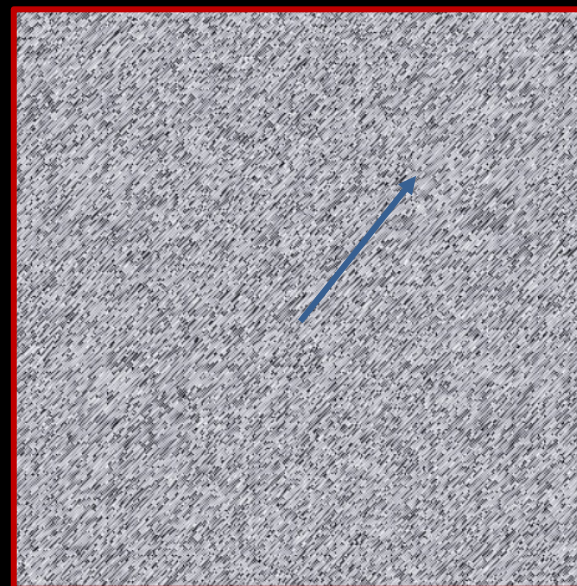
Amplitude



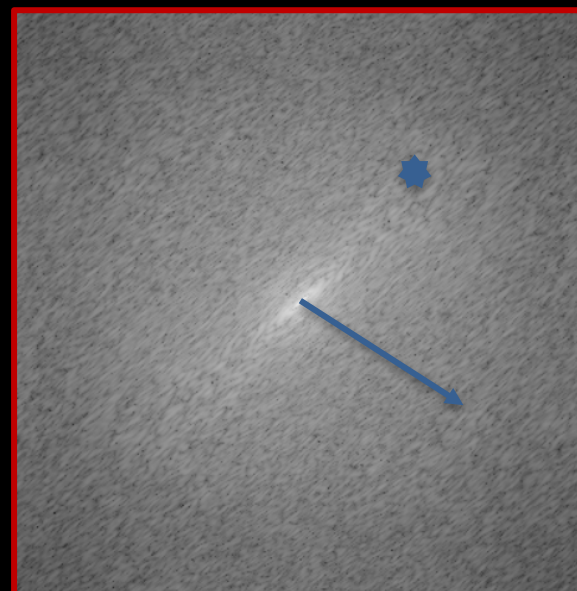
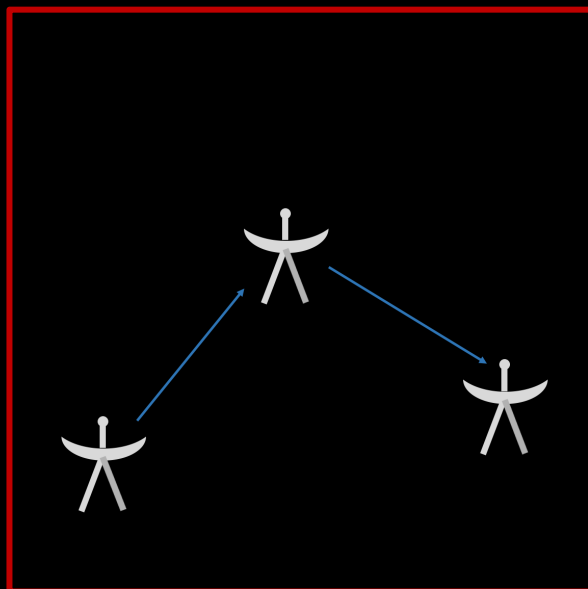
Phase



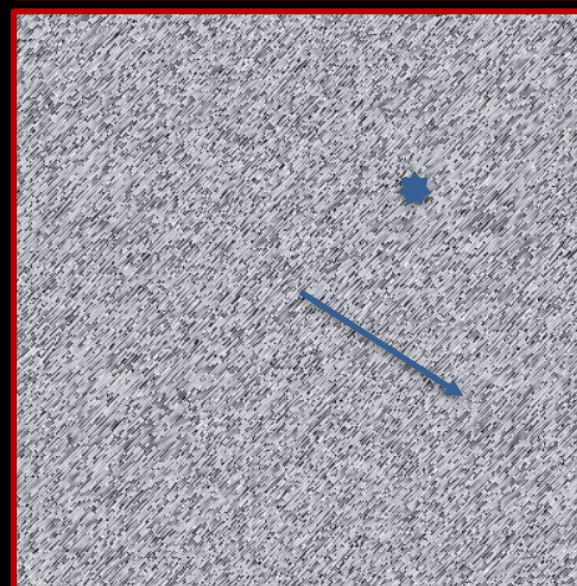
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Phase

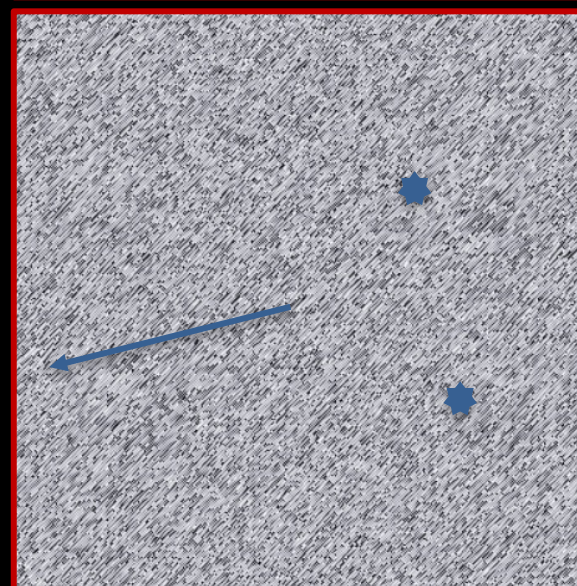
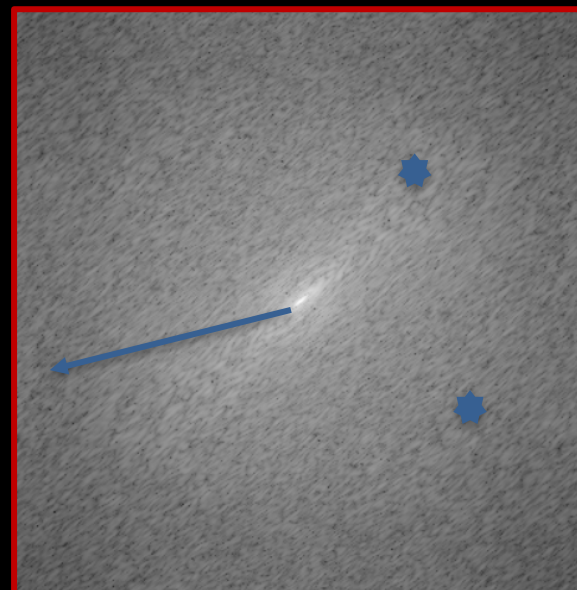
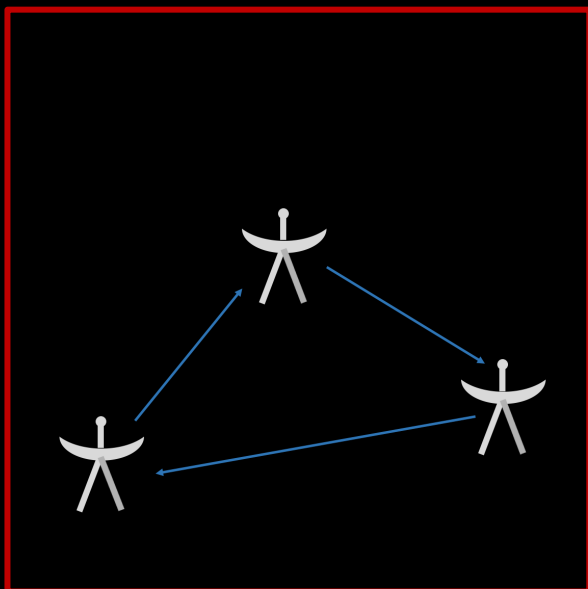


Amplitude

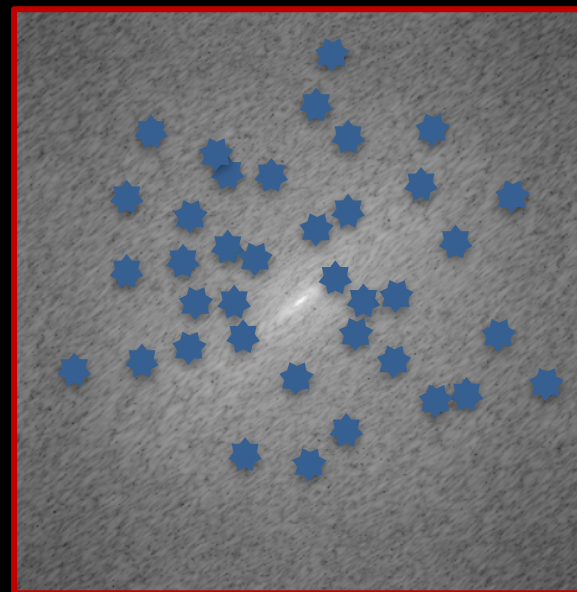
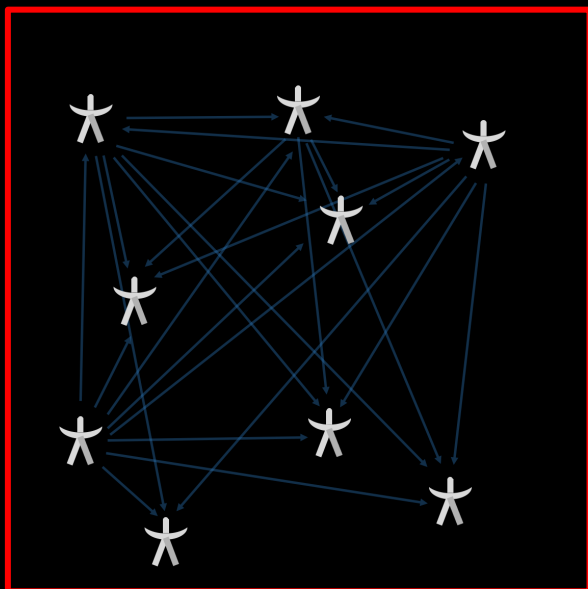


Phase

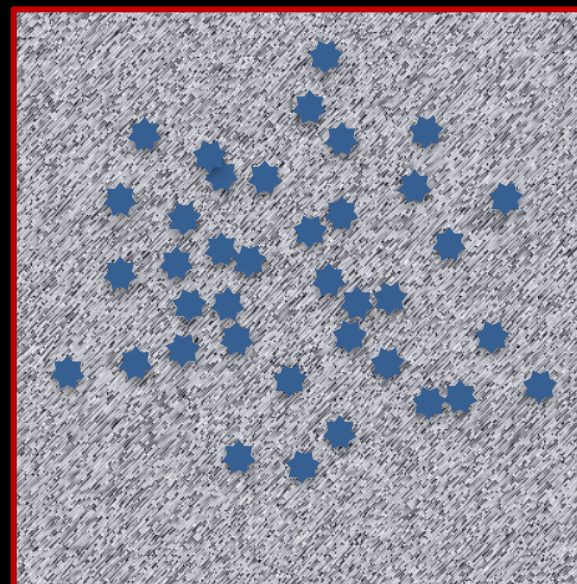




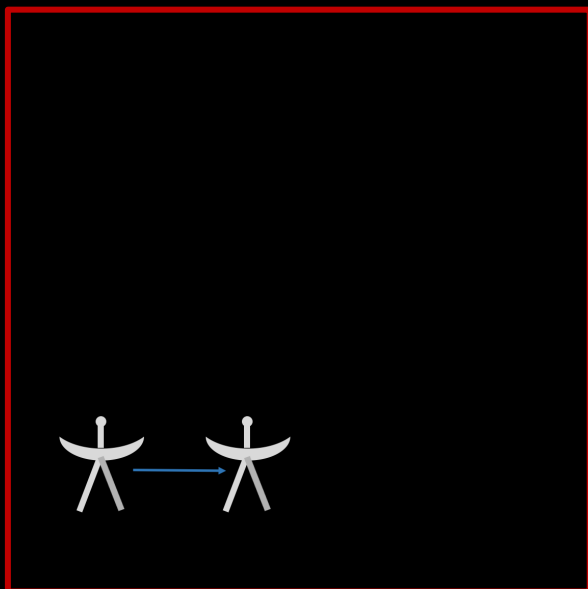




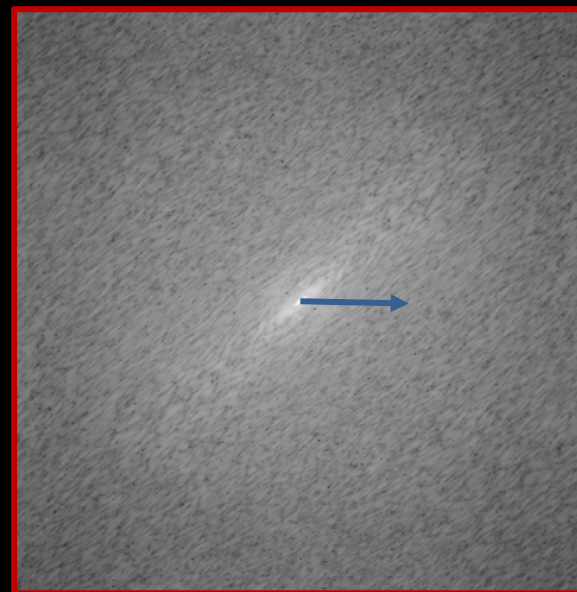
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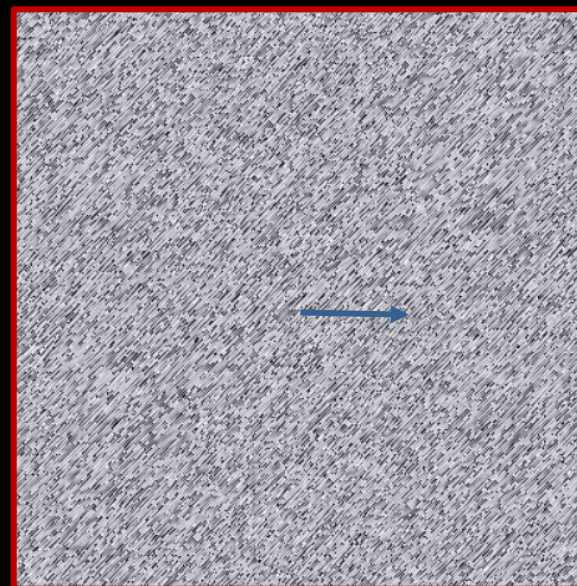
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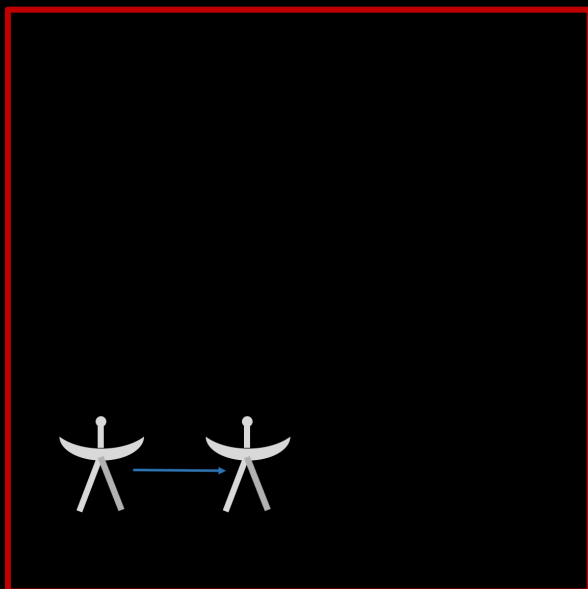
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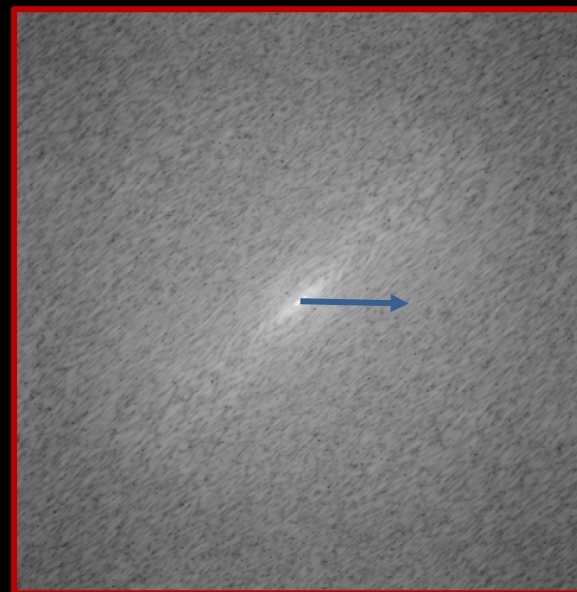
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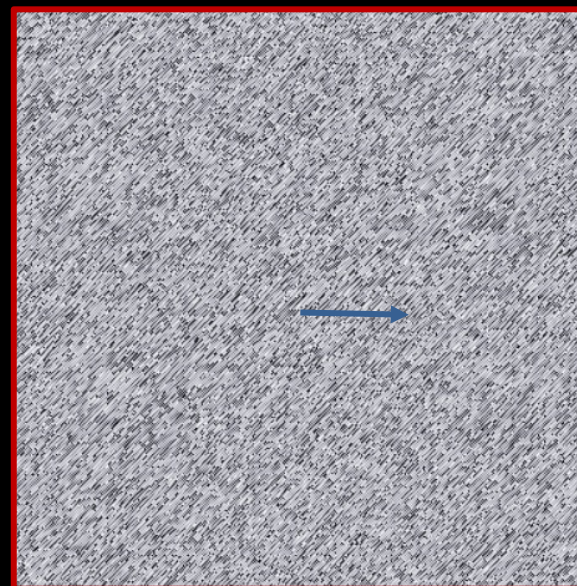
Phase



$$g_1 g_2 v$$

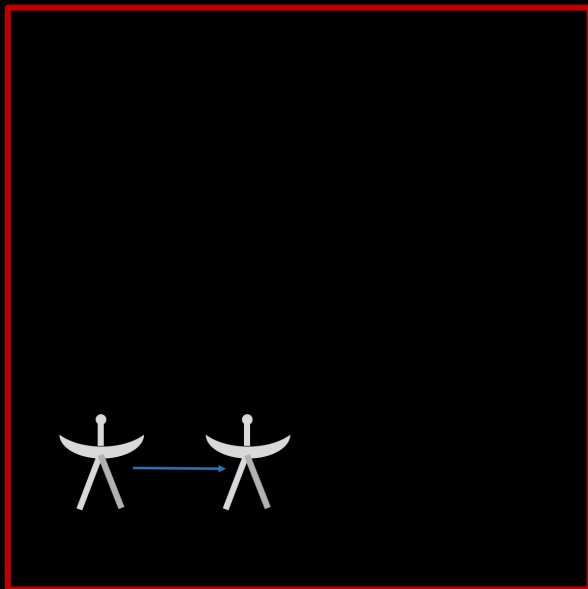


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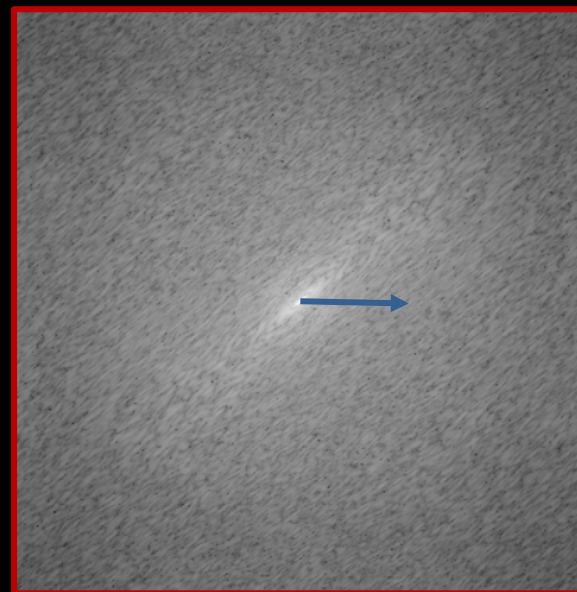


Phase

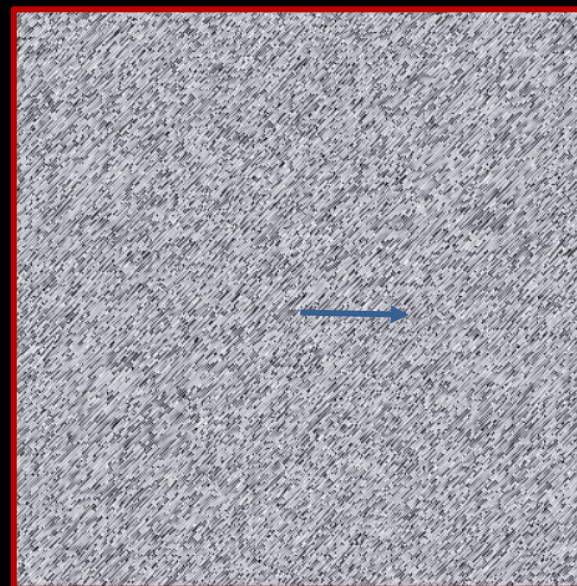




$$c_{12} = g_1 g_2 v$$



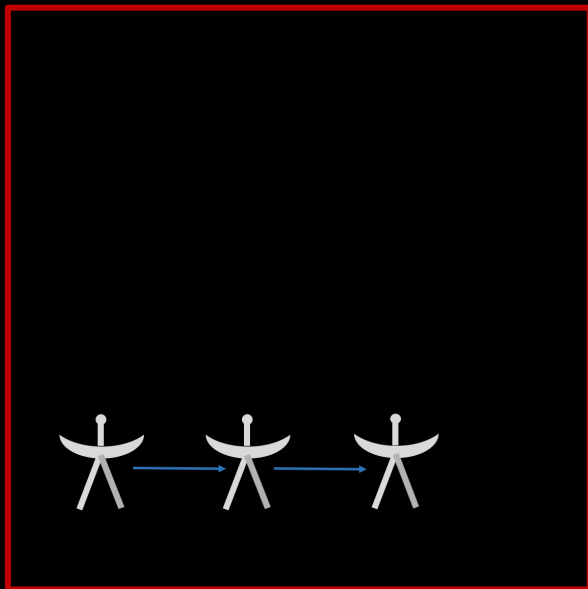
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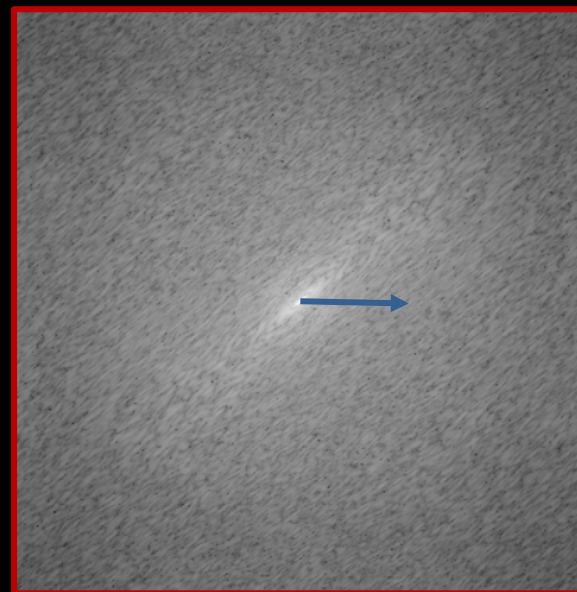
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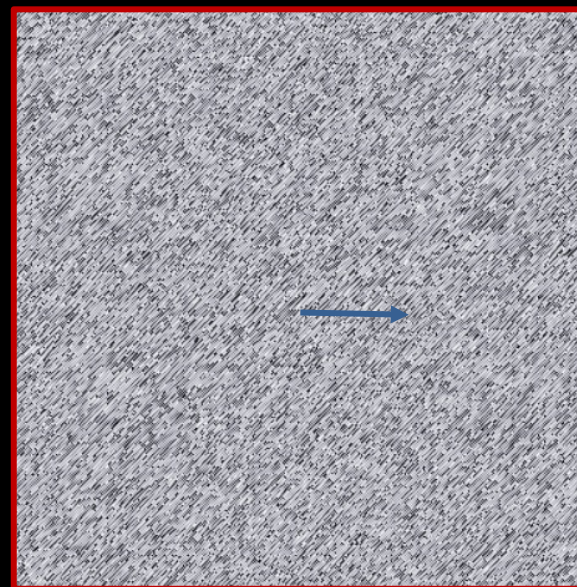
# Redundant Calibration



$$c_{12} = g_1 g_2 v$$

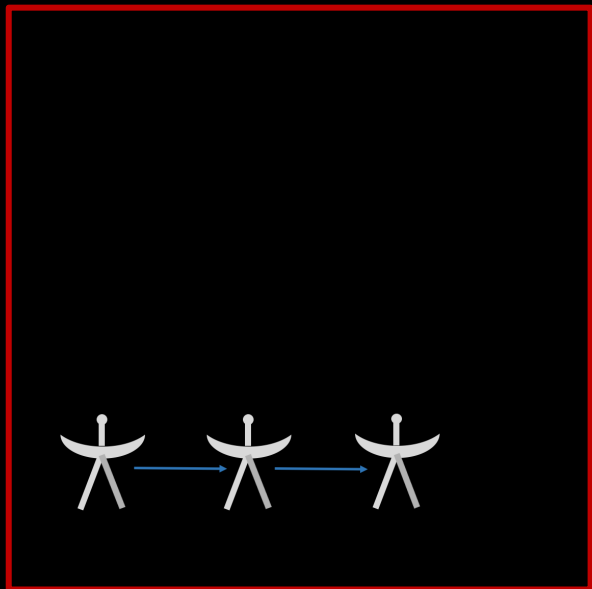


Amplitude



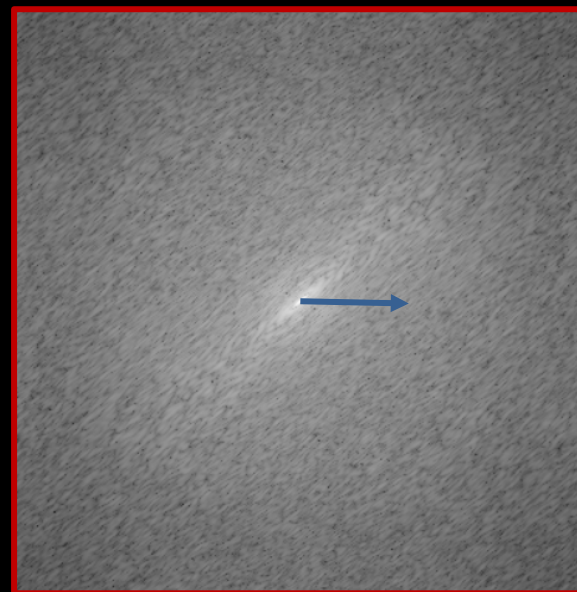
Phase

# Redundant Calibration

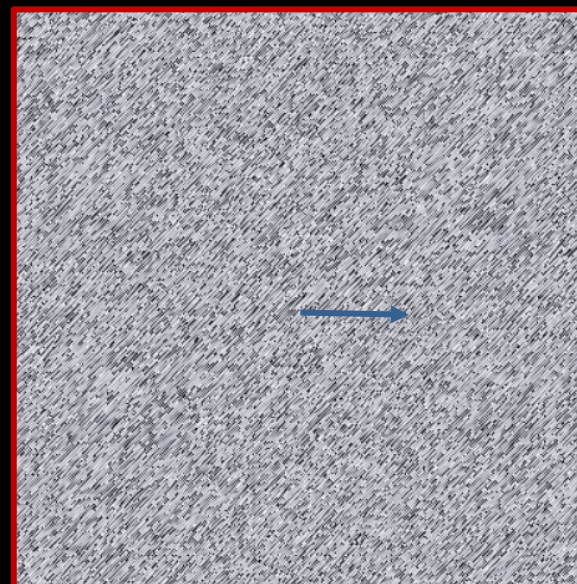


$$c_{12} = g_1 g_2 v$$

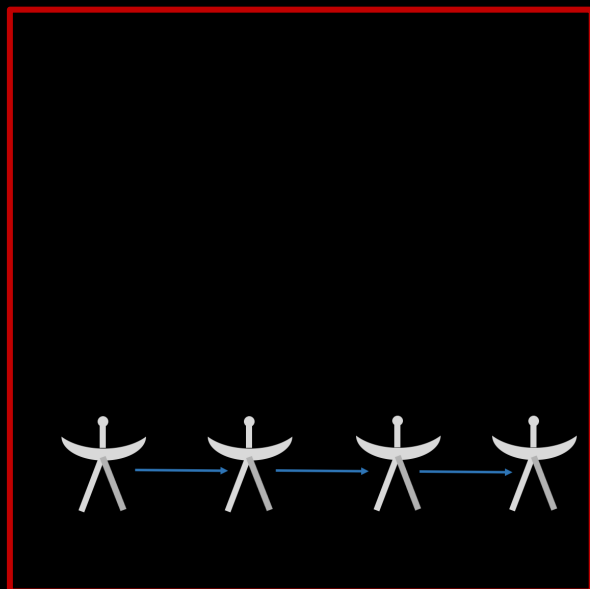
$$c_{23} = g_2 g_3 v$$



Amplitude



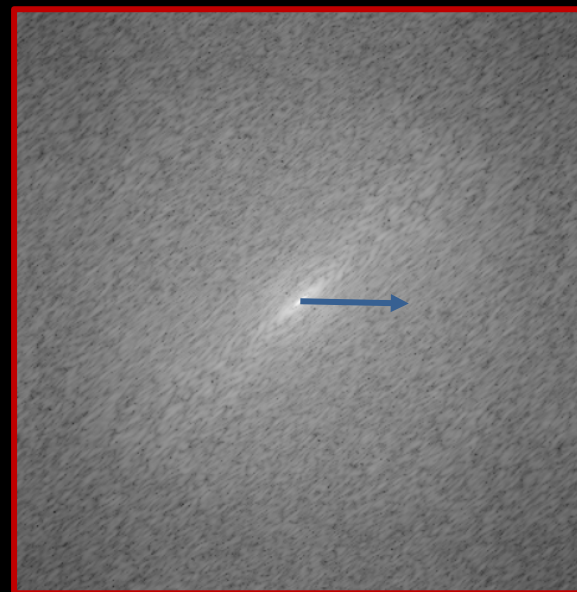
Phase



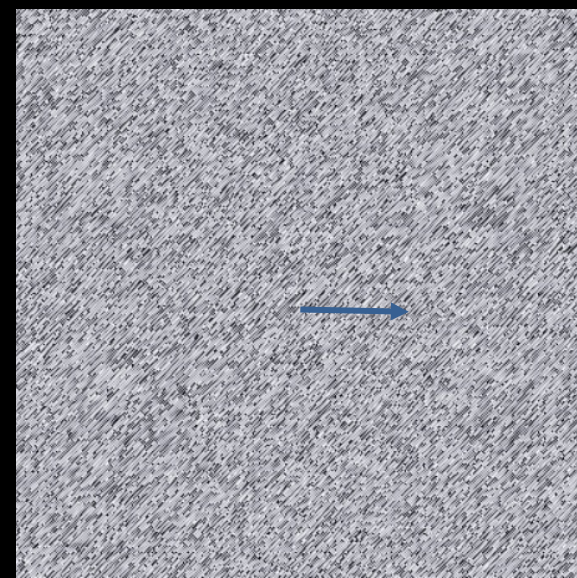
$$c_{12} = g_1 g_2 v$$

$$c_{23} = g_2 g_3 v$$

$$c_{34} = g_3 g_4 v$$



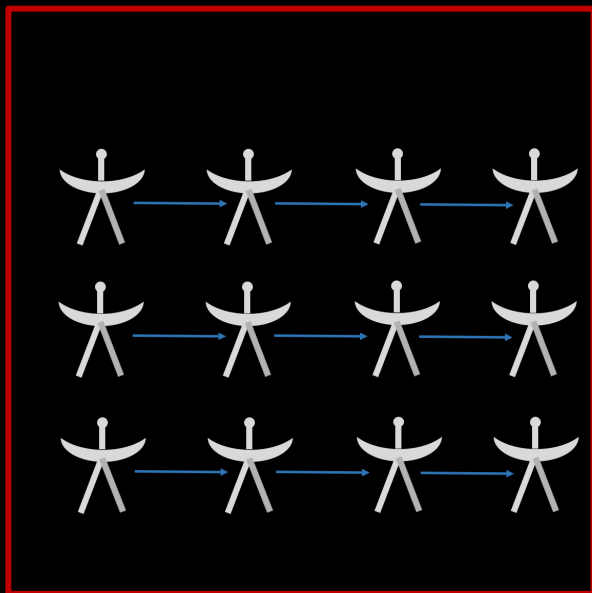
Amplitude



Phase



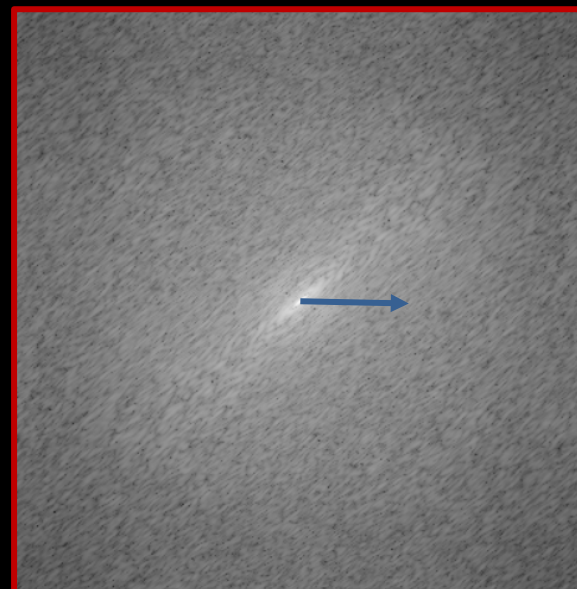
# Redundant Calibration



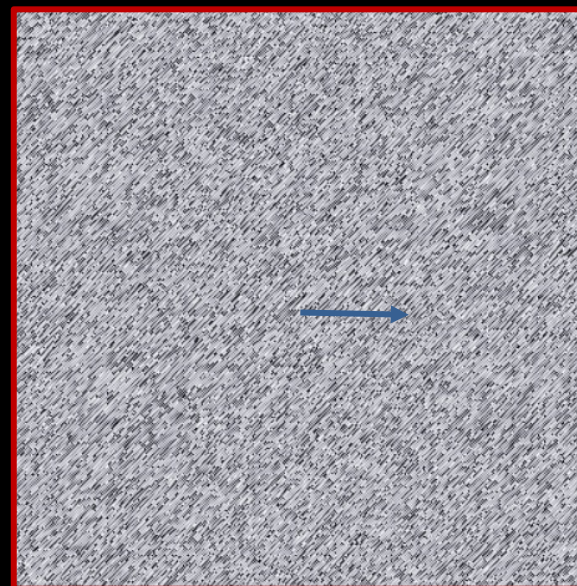
$$c_{12} = g_1 g_2 v$$

$$c_{23} = g_2 g_3 v$$

.....



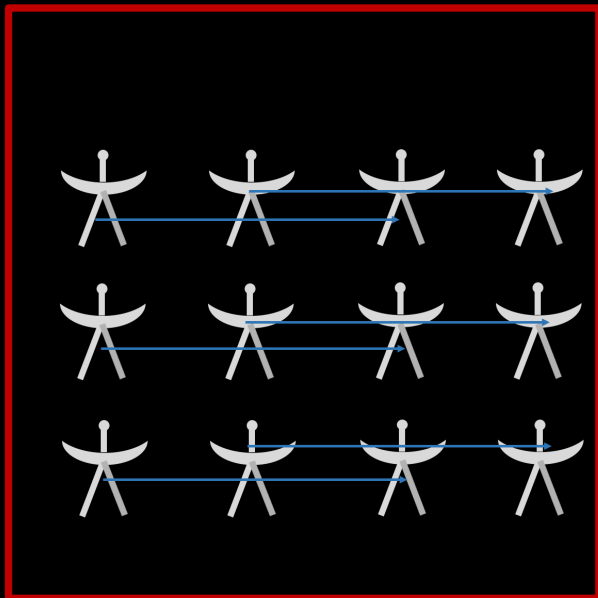
Amplitude



Phase



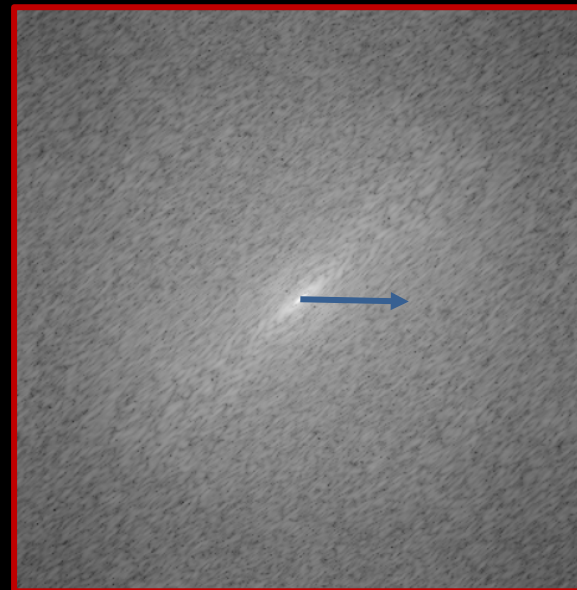
# Redundant Calibration



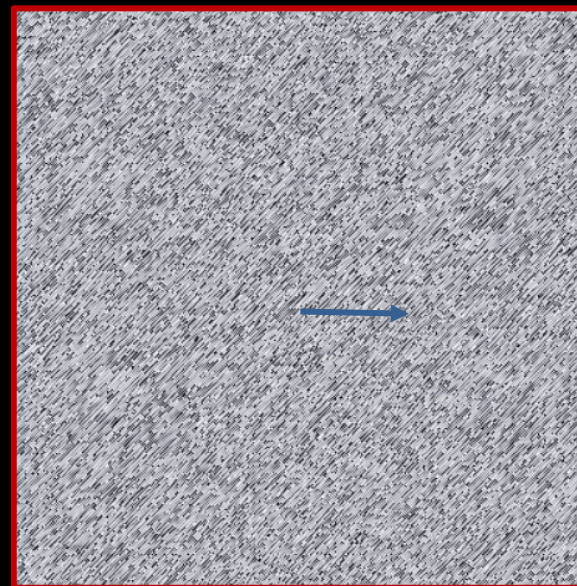
$$c_{12} = g_1 g_2 v$$

$$c_{23} = g_2 g_3 v$$

.....



Amplitude



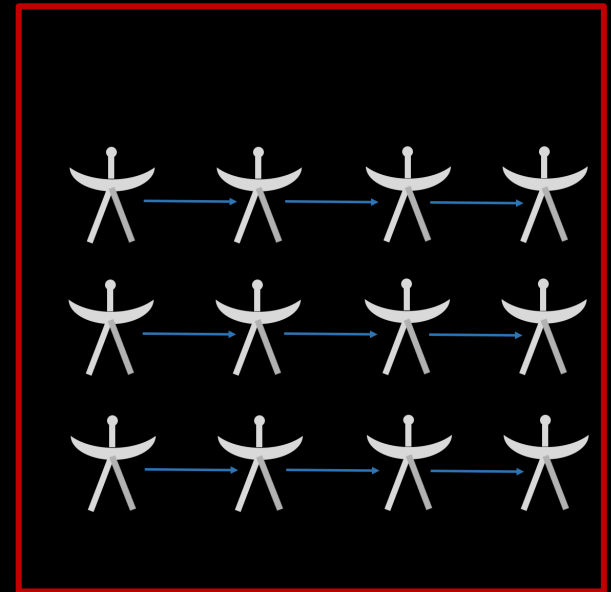
Phase

# Redundant Calibration

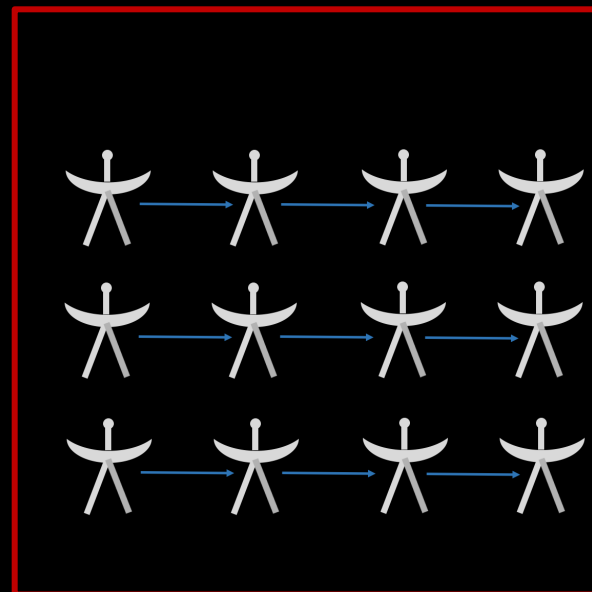
$$c_{12} = g_1 g_2 v$$

$$c_{23} = g_2 g_3 v$$

.....



$$\begin{aligned} \log c_{12} &= \log g_1 + \log g_2 + \log v \\ \log c_{23} &= \log g_2 + \log g_3 + \log v \\ \dots & \quad \dots \quad \dots \quad \dots \quad \dots \end{aligned}$$



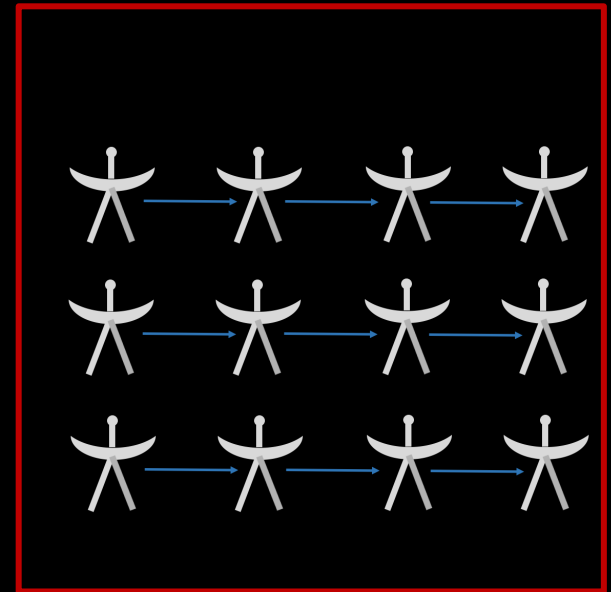
System of **linear** equations

Wieringa 1992, Liu et al. 2010



# Redundant Calibration

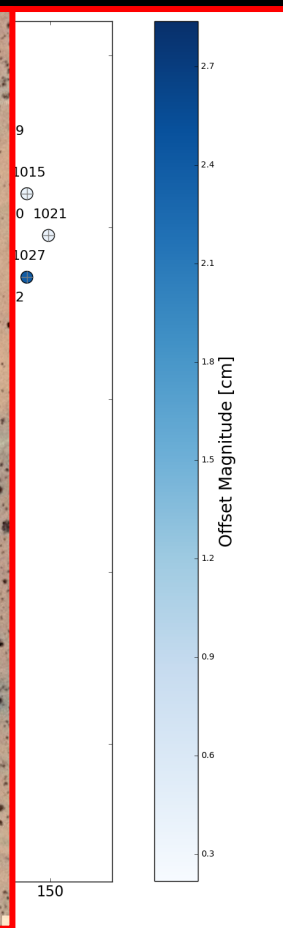
$$\begin{aligned} \log c_{12} &= \log g_1 + \log g_2 + \log v \\ \log c_{23} &= \log g_2 + \log g_3 + \log v \\ \dots & \quad \dots \quad \dots \quad \dots \quad \dots \end{aligned}$$



System of **linear** equations – Fast  
 No assumptions – Sky Model Unbiased

Wieringa 1992, Liu et al. 2010

- Real Telescopes are **not perfectly** redundant



Joseph et al. in prep.

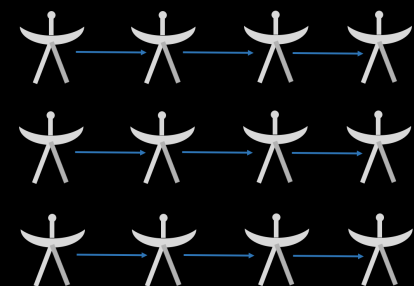
# Caveats

- Real Telescopes are **not perfectly** redundant
- Phase Wrapping

$$\log c_{12} = \log g_1 + \log g_2 + \log v$$

$$\log c_{23} = \log g_2 + \log g_3 + \log v$$

...      ...      ...      ...      ...



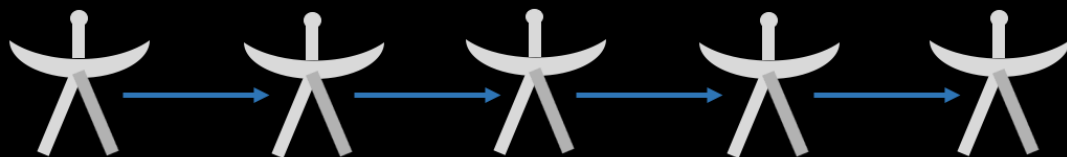
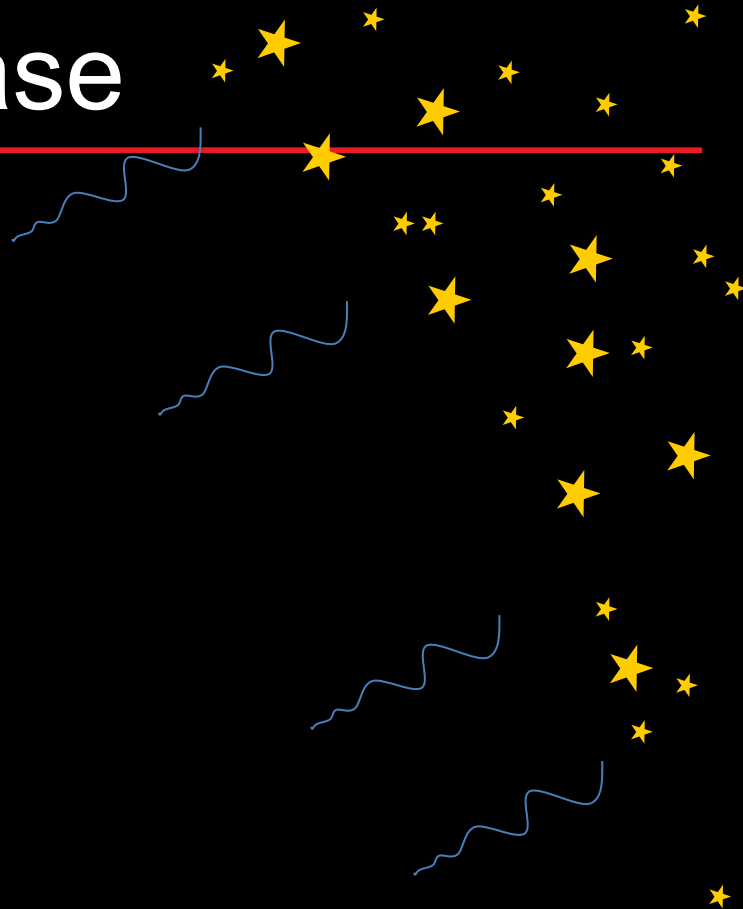
Amplitude

Phase

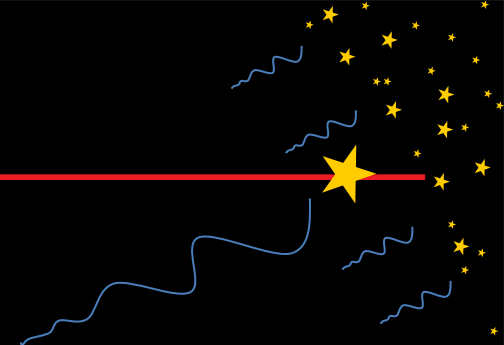




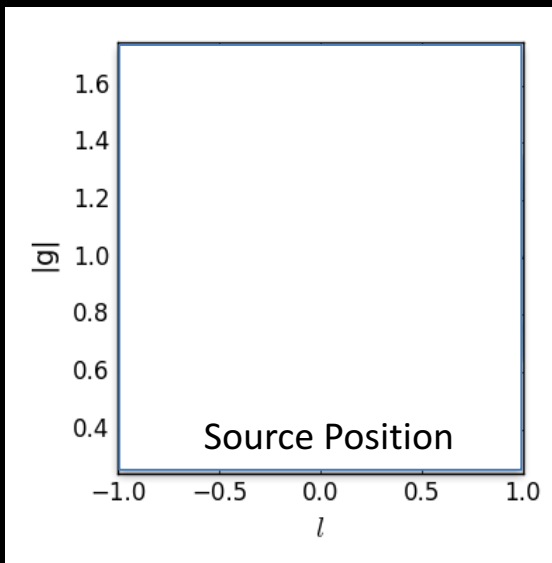
# 5 Element Test Case



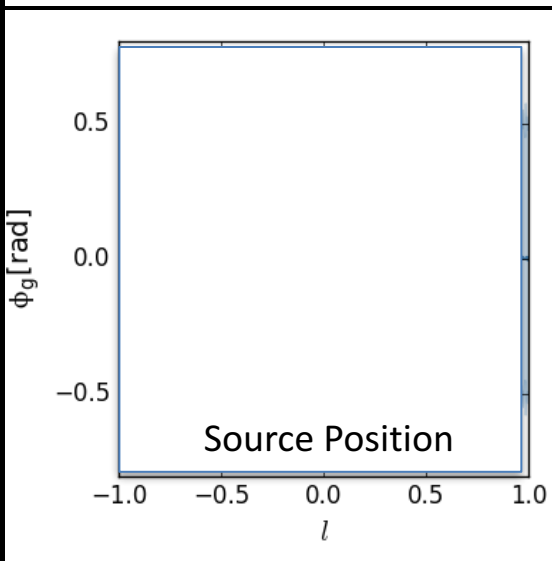
# 5 Element Test Case



Amplitude Solutions

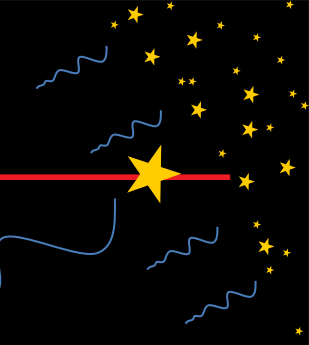


Phase Solutions

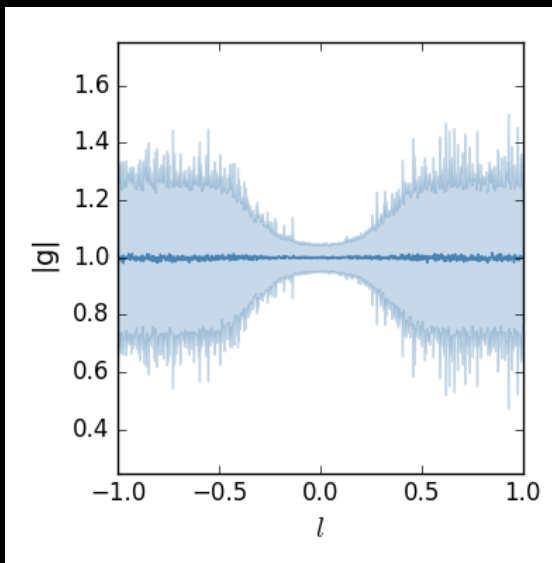


- Solution Variance follows S/N

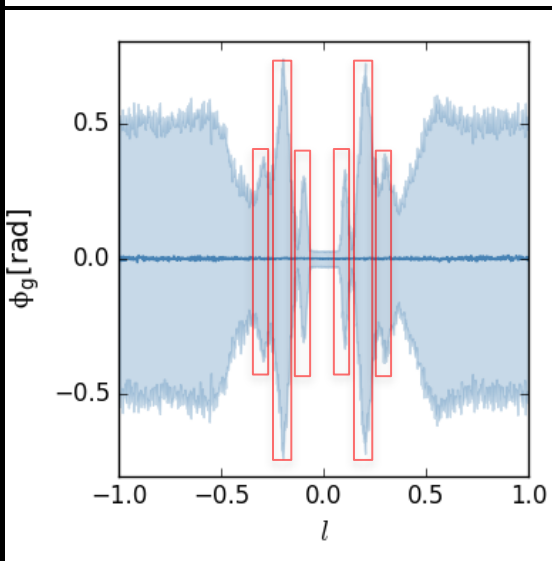
# 5 Element Test Case



Amplitude Solutions



Phase Solutions



- Solution Variance follows S/N
- Peaks caused a set of phase wrapping baselines
- Visibility phase  $\phi = \pi$
- Noise diverges  $\rightarrow 2\pi$

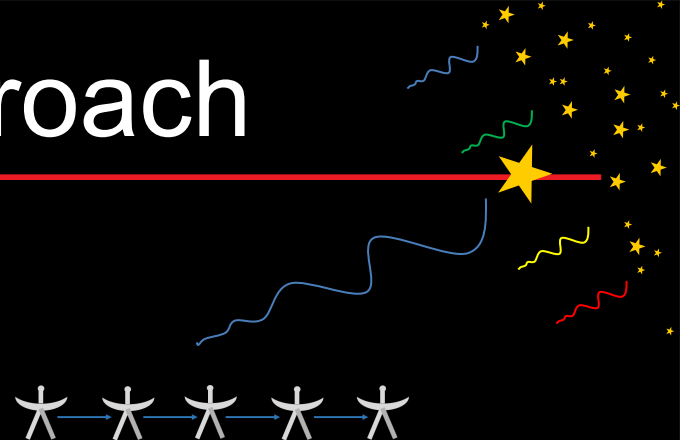
Joseph et al. in prep.





# Multi-frequency Approach

- Simultaneous solutions at different frequencies
- Baseline length  $\sim x/\lambda$



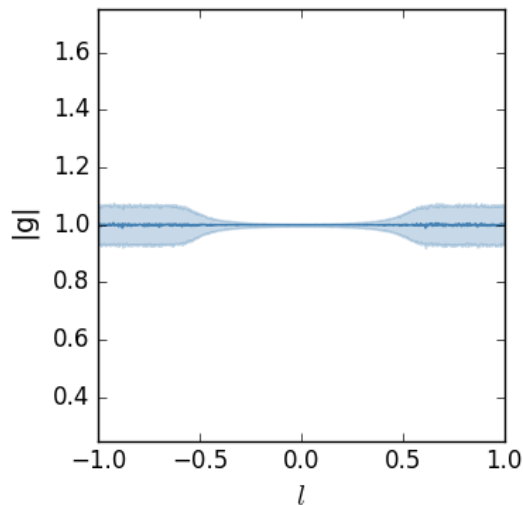
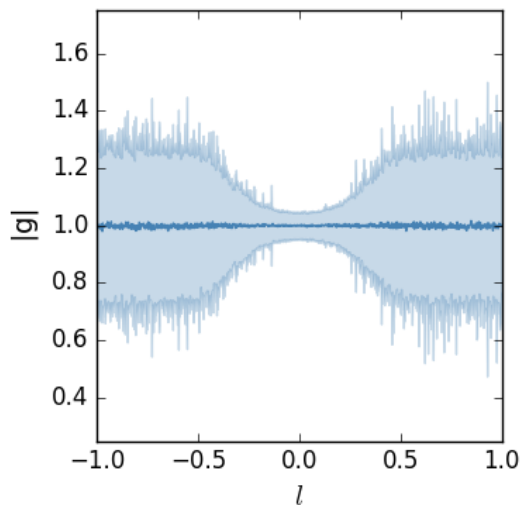


# Multi-frequency Approach

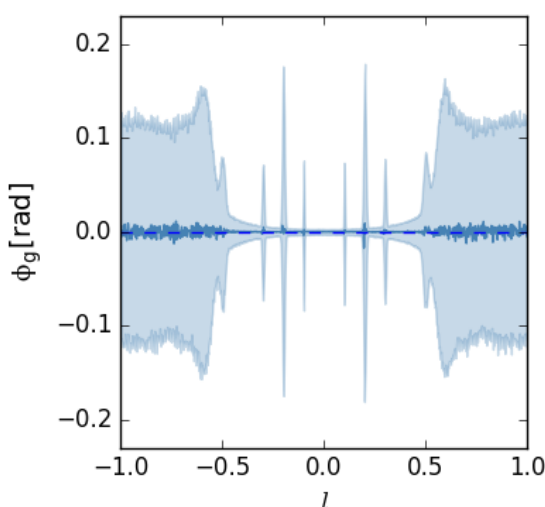
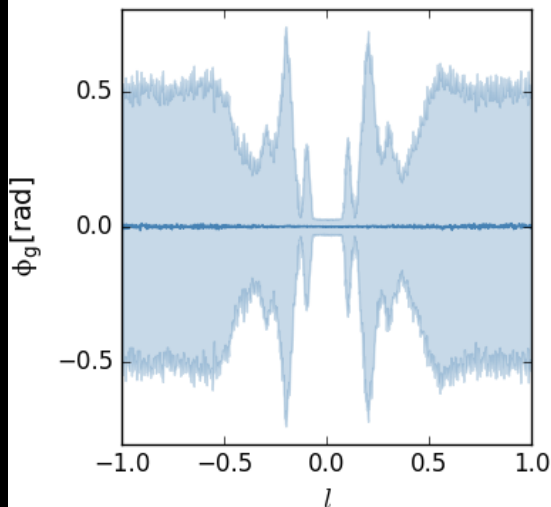
Single Frequency

Multi Frequency

Amplitude Solutions



Phase Solutions

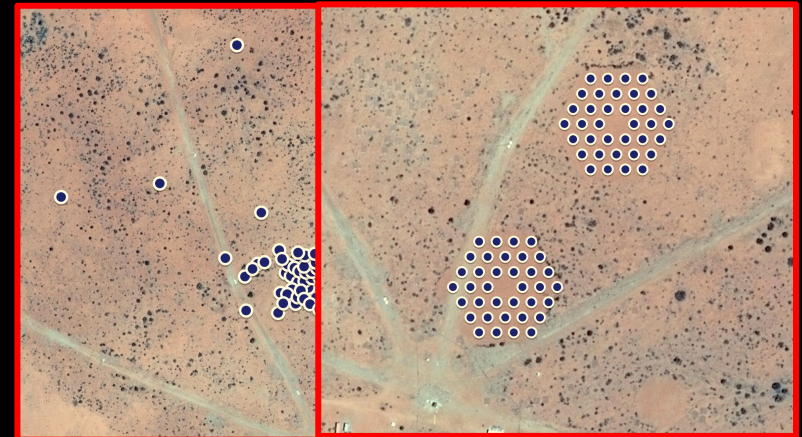


Joseph et al. in prep.

- **Model independent solutions**
- **Sky dependency**
- (Non-redundancy)
- Multi channel suffers from phase wrapping

## Possible Solutions

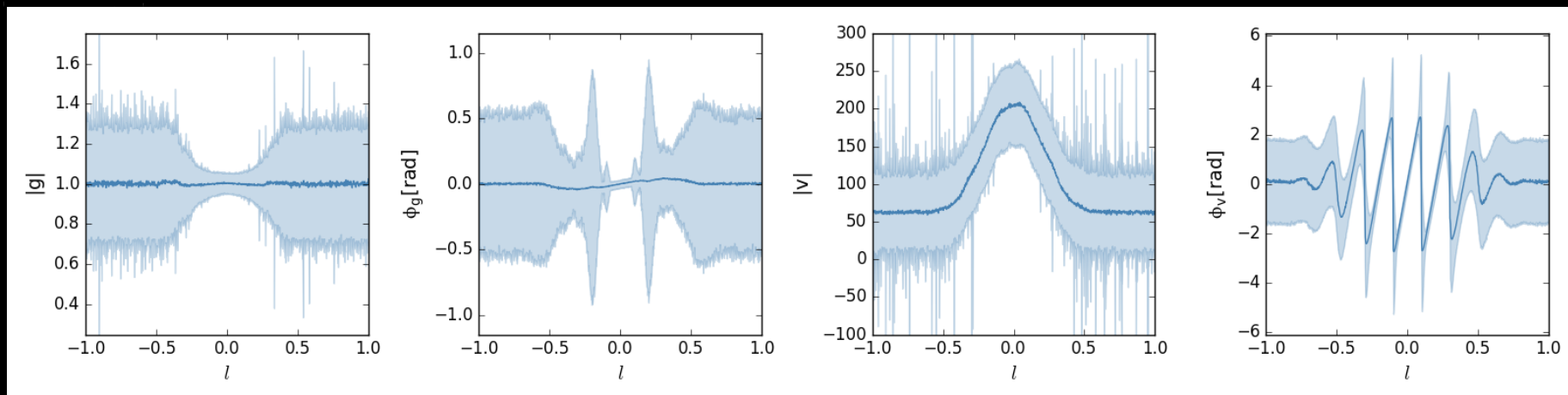
- **Hybrid Calibration**



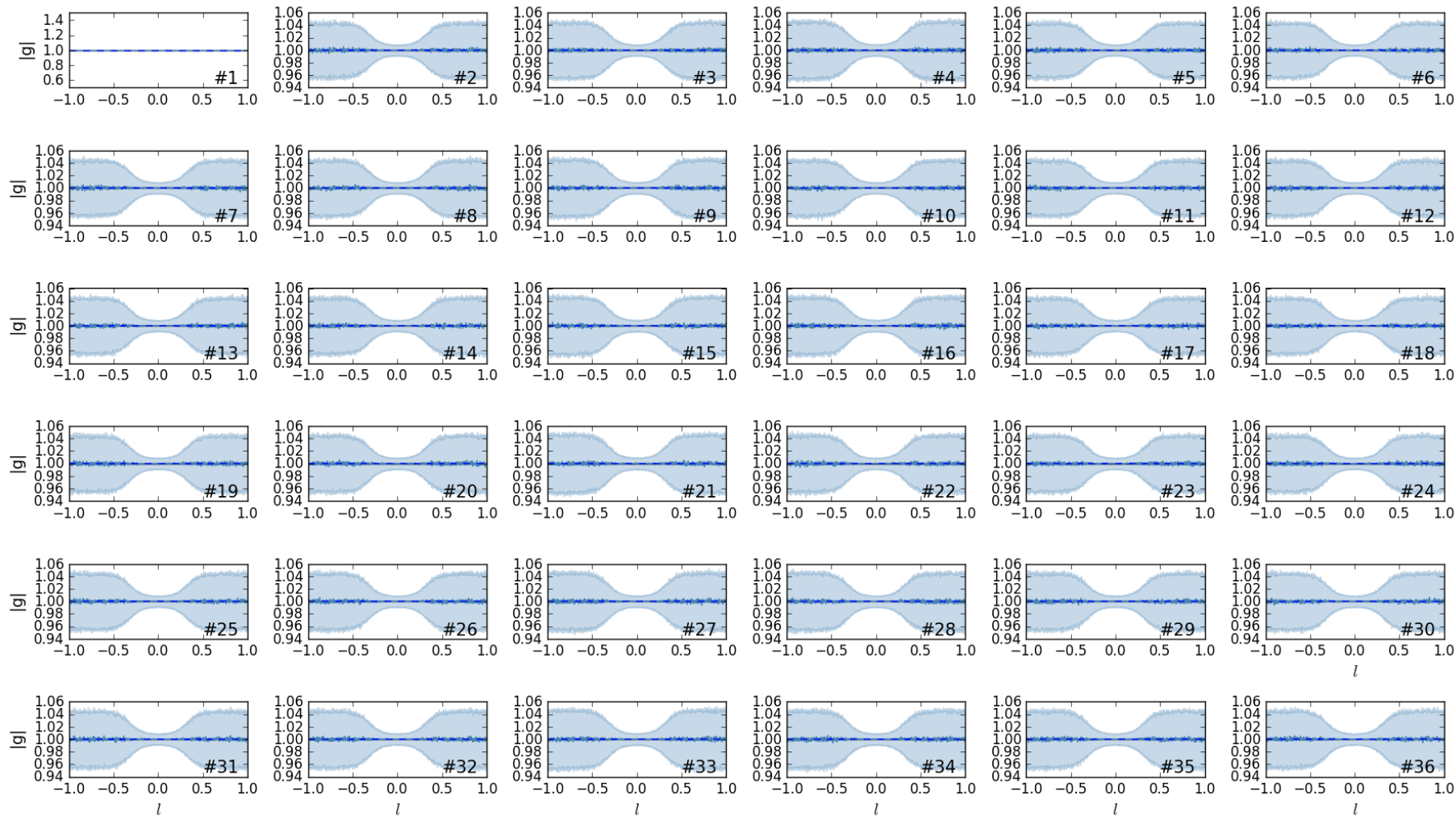
Google Maps







# Amplitude – MWA positions



# Phase – MWA Positions

